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Crack sharpness effects in fracture testing of polymers

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Tests on five polymers are described in which the fracture toughness, K_b, was determined in three-point bending using cracks with a range of tip radii. The variation of K_b with tip radius is modelled using a two criterion elastic model, a stress and a length, and using these it is possible to estimate the sharp crack values and the effects of blunting arising from the plastic zone. A suggestion for a possible standard test is given.

1. Introduction

This paper is a continuation of work previously reported [1, 2], where the influence of size on fracture toughness determined in bend and tension tests was studied. It was found that both the specimen thickness and width (depth) had to be above a critical minimum for valid plane strain values to be obtained. It was pointed out in [2], however, that only a sharp machined notch (radius $\simeq 13\,\mu$m) was used and it was not known if there was any influence of this on the data obtained. To clarify the matter, therefore, a series of tests were performed in three-point bending with specimens of sufficiently large size to give valid values with sharp notches, but in which the tip radius was varied from $13\,\mu$m up to 1 mm. For each tip radius, K_c was determined and the tests were performed on five materials; PMMA, PVC, PA, Nylon and PP (as in the previous tests). In addition to this empirical study, the work provided the opportunity to investigate further the usefulness of an elastic model for blunt cracks, which has been employed previously [3–5], and to extend its use to the plastic collapse condition which can occur for large tip radii. Finally, a procedure is suggested for a possible standard test to determine a minimum fracture toughness.

2. Analysis of blunt cracks

This analysis is based on the elastic solution for the stresses around the tip of an elliptical hole when subjected to a remote tension, p, normal to the semi-major axis, a. For very narrow ellipses, the stress, p_c, at a distance r_c from the notch tip may be written in terms of the tip radius, R, as [6]:

$$\frac{p_c}{p} = \frac{2a^{1/2}(R + r_c)}{(R + 2r_c)^{3/2}}, \qquad (R, r \ll a). \quad (1)$$

For a perfectly sharp crack, $R = 0$, and we have:

$$\frac{p_c}{p} = (a/2r_c)^{1/2}$$

and, thus, the usual expression for K_c:

$$K_c = p(\pi a)^{1/2} = p_c(2\pi r_c)^{1/2}. \quad (2)$$

For a blunt crack, we can invoke an apparent K, K_b, which gives the same p_c at the same r_c, so that we may write:

$$\frac{K_b}{K_c} = \frac{(1 + R/2r_c)^{3/2}}{(1 + R/r_c)}. \quad (3)$$

Since we are using the elastic field here, we may model contained yielding using a line zone with a stress, p_y, giving a crack opening displacement of:

$$\delta = \frac{K_b^2}{Ep_y}. \quad (4)$$

For a crack of original radius R_0, the plastic deformation characterized by δ will further blunt the tip and, if we assume smooth blunting (i.e. no discontinuities), then:

$$R = R_0 + \frac{\delta}{2}. \quad (5)$$

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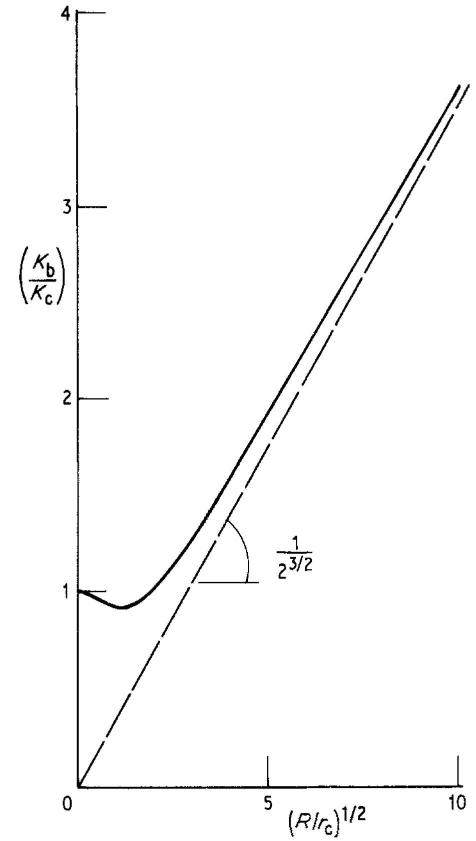

Figure 1 Crack blunting function – Equation 3.

It should be noted that fracture is defined in terms of *two* criteria here, p_c and r_c, which it is not necessary to separate for sharp cracks. For blunt cracks, it is the comparison of R and the length factor which determines the influence of the crack tip radius. In a previous paper [5], it was observed that the two criteria are most likely p_c and the crack opening displacement for a sharp crack, δ_0, given by:

$$\delta_0 = \frac{K_c^2}{Ep_y}, \tag{6}$$

but, in this study, E and p_y are constant for each condition and it is more convenient to use r_c as the length factor, but noting that

$$r_c = \frac{\delta_0}{2\pi}\frac{Ep_y}{p_c^2}. \tag{7}$$

The variation of K_b/K_c with $(R/r_c)^{1/2}$, as given by Equation 3, is shown in Fig. 1. The root form is used, since for $R/r_c \gg 1$ we have

$$\frac{K_b}{K_c} \rightarrow \tfrac{1}{2}(R/2r_c)^{1/2},$$

and the slope of the line K_b against $R^{1/2}$ enables r_c to be estimated when K_c is found from the intercept. It should be noted that K_b/K_c first decreases as R increases, giving a minimum of 0.918 for $R/r_c = 1$ and that $K_b/K_c = 1$ again for $R/r_c \simeq 3.2$.

There are some complications in actual experiments since K_b is determined for a range of R_0 values, but since δ can be found from K_b then R may be computed using Equations 4 and 5 to give K_b against $R^{1/2}$, from which K_c and r_c may be found using Equation 3. A systematic method is to take the results of any two tests for which we have values K_{b1} and K_{b2} from R_1 and R_2, and from Equation 3 we have two equations:

$$K_{b1} = \frac{K_c}{2(2r_c)^{1/2}}\frac{(2r_c+R_1)^{3/2}}{(r_c+R_1)},$$

and similarly for K_{b2} and R_2. By dividing these two equations, we have an iterative scheme for finding r_c:

$$r_c = \frac{1}{2}\left(\frac{AR_2 - R_1}{1-A}\right),$$

$$A = \left[\frac{K_{b1}}{K_{b2}}\left(\frac{r_c+R_1}{r_c+R_2}\right)\right]^{2/3} \tag{8}$$

and hence K_c. (The method will be discussed in more detail in Sections 3 and 4.)

It should also be noted that there is a theoretical minimum K_b value, $\bar{K}_b$, which it is possible to obtain because of the coupling in Equations 3, 4 and 5. The true minimum on Fig. 1 is obtained for

$$R_0 = r_c - \frac{27}{64}\delta_0,$$

giving $\bar{K}_b = 0.918K_c$ but for $\delta_0/2r_c > 32/27 = 1.185$, then that for $R_0 = 0$ is the lowest value and is obtained from the solution of

$$\frac{\bar{K}_b}{K_c} = \frac{[1+(\bar{K}_b/K_c)^2(\delta_0/2r_c)/2]^{3/2}}{[1+(\bar{K}_b/K_c)^2(\delta_0/2r_c)]}$$

and Fig. 2 shows this function. Note that for $\delta_0/2r_c \rightarrow 8$, the analysis suggests that $\bar{K}_b$ will be large, i.e. ductile behaviour, and the criterion for this condition can be written as

$$\frac{\pi}{8}\frac{p_c^2}{Ep_y} = 1. \tag{9}$$

A further complication in the tests is that as K_b increases with increasing R_0, so does the stress

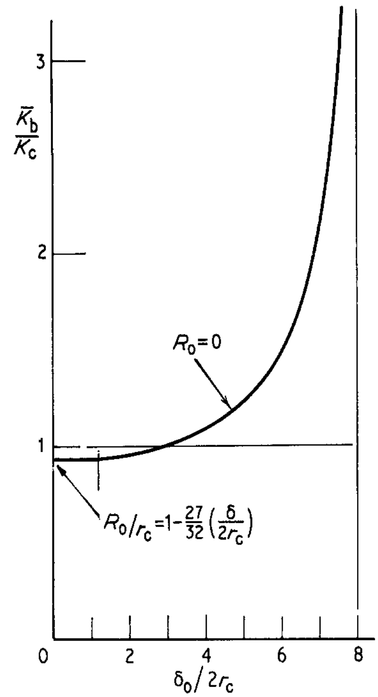

Figure 2 Minimum achievable K_b value.

level at failure and thus a plastic collapse condition can be induced in the uncracked specimen ligament (the width effect [1, 2]). It has been shown that this can be modelled quite well [1, 2] using the line zone model. The effect of higher stress may be described by

$$K_{b,el} = \frac{2}{\pi}\hat{K}\left[\ln\left(\sec\frac{\pi}{2}\frac{K_b}{\hat{K}}\right)^2\right]^{\frac{1}{2}} \quad (10)$$

where $\hat{K}$ is the plastic collapse value of K_b for a given ratio of crack length to width, and $K_{b,el}$ is the true (corrected) K_b value. For the three-point bend test used here, we may write

$$\hat{K} \simeq 2(a/W)^{1/2}W^{1/2}p_u, \quad (11)$$

where p_u is the equivalent elastic stress at the collapse condition. For first yield, we have

$$p_u = p_y\left(1-\frac{a}{W}\right)^2, \quad (12)$$

and at full plasticity, we may replace p_y by $1.5p_y$. (No constraint factor is included here.)

It is necessary to keep the specimen thickness within the criterion $B \geqslant 2.5(K_b/p_y)^2$ to maintain sufficient constraint. The width criterion, $W \geqslant 5 (K_b/p_y)^2$, [1, 2] is very difficult to sustain and the line zone correction is invoked to compensate for this.

3. Experiments and results

The five polymers chosen were tested at temperatures which gave brittle fractures in three-point bend tests using a notch tip radius range of 12.5 μm to 1 mm. All the specimens had $a/W = 0.3$ and the other test conditions are listed in Table I. The modulus value E given in Table I was calculated from an unnotched bend test and the yield stress was obtained in simple tension. For each fracture test, a minimum of five specimens were tested and K obtained using the usual calibration factors. Details of the experimental method can be found in [2].

The experimental results for all five materials are shown in Fig. 3 as K_b against $R_0^{1/2}$ and, for all but PMMA, there is a clear fully plastic condition. If this value $\hat{K}$ is plotted versus $p_y W^{1/2}$, we have the result shown in Fig. 4. The collapse conditions given in Equations 10 and 11 give

TABLE I

Material	B (mm)	W (mm)	S/W	$\dot{X}$ (cm min^{-1})	T (° C)	p_y (MPa)	E (GPa)
PMMA	20	9.0	8	0.5	20	81	2.94
PVC	20	12.5	8	0.5	20	58	3.15
PA	20	20.0	4	0.1	20	68	2.62
Nylon	20	20.0	4	0.1	−40	111	2.56
PP	20	20.0	4	0.1	−60	70	2.10

PMMA : ICI Perspex, cast sheet (9 mm)
PVC : ICI unplasticized Darvic 110 (12.5 mm)
PA : DuPont Delrin, extruded sheet (21 mm)
Nylon : modified ICI Nylon 66
PP : polypropylene copolymer, ICI extruded sheet
B = specimen thickness, S = span, $\dot{X}$ = cross-head rate, W = specimen width, p_y = tensile yield strength and T = test temperature.

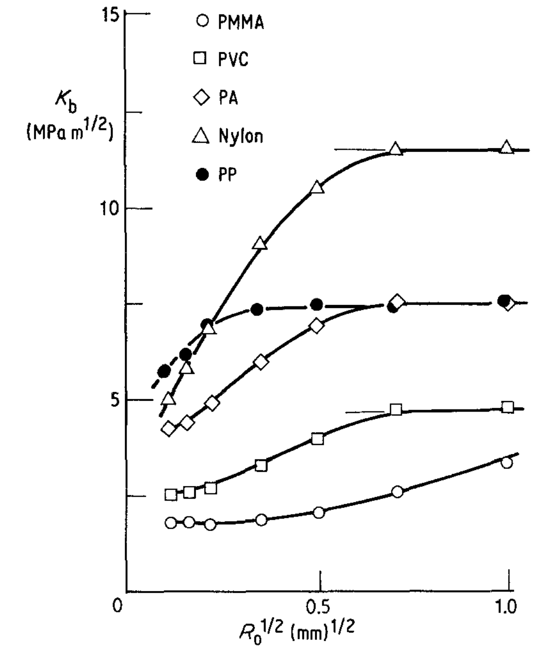

Figure 3 K_b as a function of original notch tip radius.

$$\hat{K} = \left(\frac{1}{1.5}\right)\left[2(a/W)^{1/2}\left(1-\frac{a}{W}\right)^2\right]p_y W^{1/2}$$

$$= \left(\frac{1}{1.5}\right)0.54 p_y W^{1/2}$$

for $a/W = 0.3$, and these two lines are also shown in Fig. 4. The fully plastic condition is clearly close to the observations and a factor of 1.37 describes the data accurately. This value was used to define a $\hat{K}$ for PMMA and then all the K_b values were corrected using Equation 10 to find $K_{b,el}$. The increases in K_b were around 10 to 20%, except for PP for which much larger values occurred.

The $K_{b,el}$ values were then used to calculate δ, and R was found from Equation 5 giving the $K_{b,el}$ against $R^{1/2}$ data shown in Fig. 5. The lines shown were fitted using the iteration method given in Equation 8. Pairs of values were chosen from which r_c and hence K_c could be found. Matching the lower and higher points gave average values and the parameters are given in Table II, together with p_c and δ_0. PMMA presents special problems in that the δ_0 value is much too large [7] and should be about 2 μm. This arises because the K values are at instability where the crack is moving, and the E and p_y values should each be about a factor of two higher. The δ corrections used are thus much too large but they make very little difference here since the increase in K is not large. In fact, the large r_c value is about the craze length and p_c comparable to craze stress values, indicating that p_c and r_c are describing the craze here. $\delta_0/2r_c$ is also given in Table II and for PMMA is low, suggesting a K_b minimum value of about $0.92K_c$ (see Fig. 2). For PVC, r_c is somewhat smaller, giving an increased p_c value and a constraint factor (p_c/p_y) of about 5. $\delta_0/2r_c$ is only just greater than

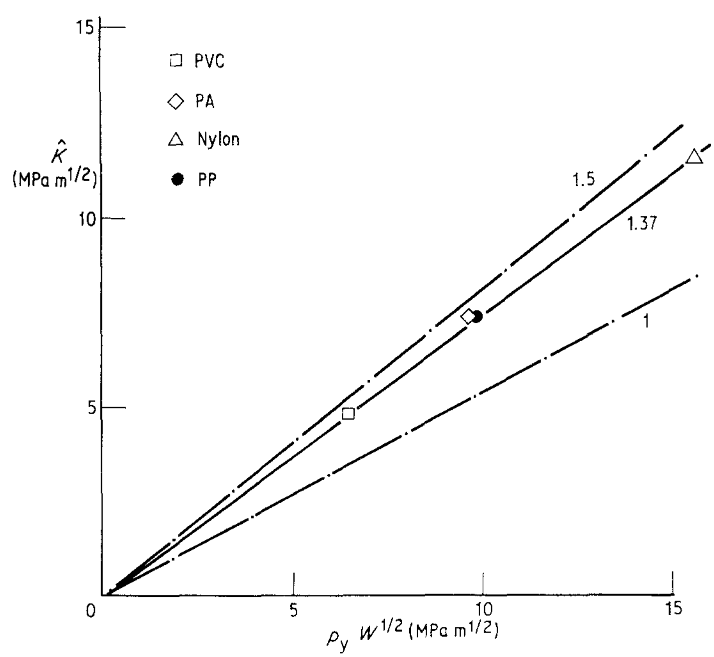

Figure 4 Plastic collapse $\hat{K}$ as a function of $p_y W^{1/2}$ for $a/W = 0.3$. Lines from Equations 10 and 11.

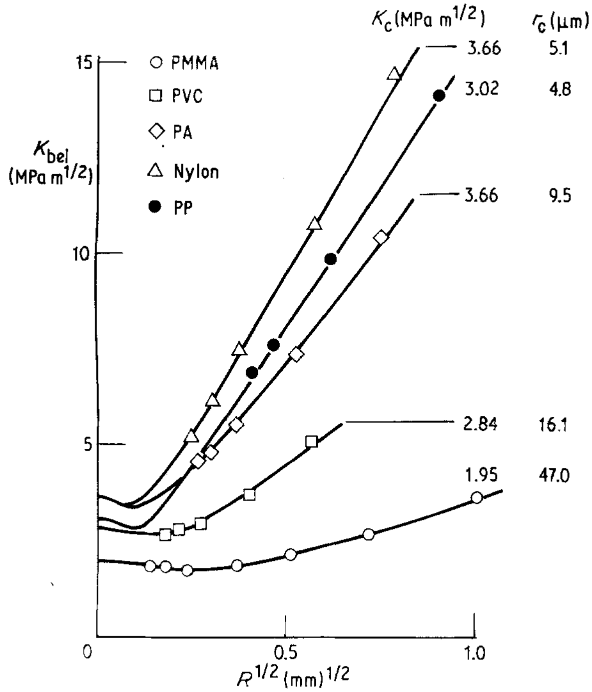

Figure 5 Corrected K_b values plotted against total root tip radius. Lines are fitted from Equation 3.

unity, so again $K_b < K_c$. The other three materials all have low r_c values and constraint factors in the range 6 to 8 with high values of $\delta_0/2r_c$, so $\bar{K}_b$ is greater than K_c. Most notable is PP, where there is substantial blunting above the sharp notch value.

It is interesting to compare the $\bar{K}_b$ values with those obtained at the minimum crack values of 12.5 μm shown in Table II. The values for PMMA and PVC are actually *below* $\bar{K}_b$ because of the decrease from the plasticity effects, but the differences are small (≃ 5%). The measured values for the other three materials are all 10 to 20% higher, but this is helped by the effects of plasticity which suppresses the values, thus giving some compensation for the bluntness of the notch. It is worth noting that $\bar{K}_b$ is the minimum value at a given temperature but K_c does not change greatly with temperature, so that at lower temperatures when E and p_y are higher, $\bar{K}_b$ would probably tend to K_c and this value is a safer minimum since it ignores the beneficial effect of crack blunting. The relatively poor position of PP compared with PA and Nylon on this basis is probably more representative of service conditions.

Table II also gives the parameter $(\pi/8)[p_c^2/(Ep_y)]$ $(= \delta_0/16r_c)$ and its relative value to unity is a useful indicator of degree of ductility to be expected.

4. A possible standard test

In view of the importance of the determination of a minimum K value, a procedure is suggested here as a basis for a possible standard. Before adoption, it would, of course, require careful confirmation. It is proposed that tests be performed at two notch radii; the smallest possible, which is about 12.5 μm for a very sharp cutter, and some larger value (100 to 1000 μm). For both cases, the

TABLE II Fracture data

Material	K_c (MPa m$^{1/2}$)	r_c (μm)	p_c (MPa)	δ_0 (μm)	$\frac{\delta_0}{2r_c}$	$\bar{K}_b$ (MPa m$^{1/2}$)	K_b at $R_0 = 12.5$ μm (MPa m$^{1/2}$)	$\frac{p_c}{p_y}$	$\frac{\pi}{8}\frac{p_c^2}{Ep_y}$
PMMA	1.95	47.0	113	16.0	0.17	1.87	1.80	1.39	0.02
PVC	2.84	16.1	282	44.1	1.37	2.61	2.50	4.86	0.17
PA	3.66	9.6	471	75.1	3.90	3.92	4.25	6.93	0.49
Nylon	3.66	5.1	646	47.1	4.62	4.21	5.00	5.82	0.58
PP	3.02	4.8	550	62.0	6.46	5.04	5.80	7.86	0.81

TABLE III Calculations for two radii

Material	$\hat{K}$ (MPa m$^{1/2}$)	K_{b1} at $R_0 = 12.5\,\mu m$ (MPa m$^{1/2}$)	K_{b2} (MPa m$^{1/2}$)	K_1 (MPa m$^{1/2}$)	K_2 (MPa m$^{1/2}$)	R_1 (μm)	R_2 (μm)	r_c (μm)	K_c (MPa m$^{1/2}$)	$\bar{K}_b$ (MPa m$^{1/2}$)	$2.5\left(\frac{K_{b2}}{p_y}\right)^2$ (mm)
PMMA	5.77	1.80	3.35 $R_0 = 1000\,\mu m$	1.84	3.64	19.6	1028	44.2	1.97	1.90	4.3
PVC	4.87	2.50	4.00 $R_0 = 250\,\mu m$	2.66	4.97	31.9	317.6	15.2	2.81	2.58	11.9
PA	7.22	4.25	6.00 $R_0 = 125\,\mu m$	4.63	7.52	72.7	283.7	12.1	4.05	4.24	19.5
Nylon	11.79	5.00	10.50 $R_0 = 250\,\mu m$	5.20	14.10	60.1	599.8	5.8	3.84	4.33	22.4
PP	7.43	5.80	6.90 $R_0 = 50\,\mu m$	6.97	9.90	177.7	383.4	5.9	3.37	5.69	24.3

thickness requirement

$$B \geqslant 2.5\left(\frac{K_b}{p_y}\right)^2$$

should be observed [2]. The value obtained at the higher value must be less than the collapse condition and this can be checked by using

$$\hat{K} = 2.8(a/W)^{1/2}\left(1-\frac{a}{W}\right)^2 p_y W^{1/2}.$$

Assuming both K_b values are less than $\hat{K}$, they may then be corrected using

$$K_{b,el} = \frac{2}{\pi}\hat{K}\left[\ln\left(\sec\frac{\pi}{2}\frac{K_b}{\hat{K}}\right)^2\right]^{\frac{1}{2}}.$$

The value of δ may be found from

$$\delta = \frac{K_{b,el}^2}{Ep_y}$$

and then the radius from

$$R = R_0 + \frac{\delta}{2}.$$

The resulting two values of $K_{b,el}$ and R may then be processed to find K_c and r_c, and hence $\delta_0/2r_c$ to find $\bar{K}_b$ if required.

This procedure has been applied to the data obtained here and the results are shown in Table III. The values of r_c, K_c and $\bar{K}_b$ are computed using the simple program given in Appendix 1. The values agree reasonably with those in Table II, with some differences, as expected, when only two radii are used. The largest thickness requirement, $2.5(K_{b2}/p_y)^2$, is also given and all the specimens are close to meeting this requirement.

5. Conclusion

The test method and analysis provide a useful insight into crack tip sharpness effects. As expected, the lower toughness materials (PMMA and PVC) present few problems and a wide range of initial notch values will give values close to the minimum. For the tougher polymers, the matter is more complicated because of substantial blunting and measured values may be significantly above the possible minimum values. Matters are helped somewhat by a compensating effect of plasticity offsetting the increase due to blunting, but this does not always occur. It is particularly interesting to note here that the two low toughness materials give low values for a wide range of notch tip radii, while two tough materials (PA and Nylon) have higher K_c values, as well as high K_b values. PP, on the other hand, gives high K_b values for a rather low K_c value and this is consistent with a greater tendency to brittle cracking.

Appendix

"BASIC" program for determination of blunt crack parameters: r_c, K_c, $\bar{K}_b$

K_1, R_1, K_2, R_2 — $K_{b,el}$ and R values, $K_2 > K_1$.
E, Young's modulus, p, yield stress.
Units: K in MPa $m^{1/2}$, R in μm, E in GPa, p in MPa.

```
100 INPUT"K1=,R1=,K2=,R2=,E=,P=";K1,R1,K2,R2,E,P
110 R3=1
120 R4=R3
130 A=(K1/K2*(R4+R1)/(R4+R2))↑(2/3)
140 R3=.5*(A*R2-R1)/(1-A)
150 IFABS(R3-R4)>.001THEN120
160 K3=2*K1*SQR(2*R3)*(R3+R1)*(2*R3+R1)↑(-1.5)
170 X=(1000*K3↑2)/(2*R3*E*P)
180 Y1=1
190 Y2=Y1
200 Y1=(1+.5*X*Y2↑2)↑(1.5)/(1+X*Y2↑2)
210 IFABS(Y1-Y2)>.001THEN190
220 K4=Y1*K3
230 PRINT"RC="R3;"KC="K3;"KBAR="K4
240 GOTO100
```

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